ECE 3455
Mid-Term Exam
July 13, 2010

Exam duration: 100 minutes

- You may have one 8 ½ x 11 in. “crib” sheet, written on both sides, during the quiz. You may have any calculator you choose, but no computers. No other notes or materials will be allowed.
- Show all work necessary to complete the problem on these pages. A solution without the work shown will receive no credit.
- Show units in intermediate and final results, and in figures.
- If your work is sloppy or difficult to follow, points will be subtracted.

*This exam has 10 pages, including the cover sheet. Raise your hand if you are missing a page.*

1 ______/20

2 ______/25

3 ______/25

4 ______/30

Total ______/100
1. **(20 points)** For the circuit shown below, do the following.

   i. Find the transfer function \( T(\omega) = \frac{V_o}{V_i} \).

   ii. The graph below is a magnitude Bode plot for the circuit; you can think of it as providing specifications for the circuit. Choose circuit component values to meet these specifications. Be sure your choices reproduce the important features of the Bode plot.

\[
Z_f = \frac{R_f}{1 + j\omega C_f R_f}
\]
There is something interesting going on with the output of this circuit. It would appear that the capacitors $C_i$ are in parallel, so that all we need do is:

$$\frac{R_i}{i} - \frac{C_i}{\frac{1}{j\omega C_i}} - \frac{1}{2} \frac{1}{C_i} \Rightarrow \frac{R_i}{i} \frac{2C_i}{\frac{1}{j\omega C_i}}$$

This correctly predicts the input impedance but it does not give the correct output. For that we need a more complete analysis. Looking at the original circuit:

$$\bar{V}_x = \frac{V_x}{j\omega C_i} + \frac{V_x - V_i}{R_i} = 0 \Rightarrow \bar{V}_x = \frac{V_i}{1 + j\omega C_i R_i}$$

Thus $\bar{I}_x = j\omega C_i \bar{V}_x = \frac{j\omega C_i \bar{V}_x}{1 + j\omega C_i R_i} = \bar{I}_f$

So $\bar{V}_o = -\bar{I}_x 2f = -\frac{j\omega C_i \bar{V}_x}{1 + j\omega C_i R_i} \frac{R_f}{1 + j\omega C_i R_i}$

$$\bar{T}(\omega) = \frac{\bar{V}_o}{\bar{V}_i} = \frac{-j\omega C_i R_f}{(1 + j\omega C_i R_i)(1 + j\omega C_i R_f)}$$

This is a factor of 2 less than if we had done a simple inverting configuration with the input impedance $R_i + j\omega C_i$. The reason is that the current in the feedback resistor has been reduced by half because of the capacitor to ground.
Room for extra work

In any case, C_o and R_o have no effect.

ii) From the graph we have two poles at
\[ \omega_{21} = 210 \text{ rad/s}, \quad \omega_{22} = 4300 \text{ rad/s}. \]

Our transfer function shows poles at
\[ 1/\omega_{GfR_f} = 4300 \text{ rad/s}, \quad 1/2C_0R_i = 210 \text{ rad/s}. \]

We have arbitrarily set the poles of Tiw equal to the poles indicated on the plot. The opposite assignment is also valid. So we have

\[ C_0R_f = 2.33 \times 10^{-3}, \quad R_iC_i = 2.38 \times 10^{-3}. \]

But before we assign values, we note that
\[ |T(\omega = 10^3 \text{ rad/s})| = 12.5 \text{ dB} \rightarrow 4.82 \frac{W}{V}. \]

From our transfer function, we have
\[ |T(\omega = 10^3 \text{ rad/s})| = \left| \frac{j(1000)(2.38 \times 10^{-3}) \cdot R_f}{[1+j(1000)R_i(2.38 \times 10^{-3})][1+j(1000)C_i(2.33 \times 10^{-4})]} \right| \]

\[ +3 \]
\[ = \frac{j \cdot R_f / R_i \cdot 2.38}{1+j(4.76)(1+j0)} \]

\[ = \frac{R_f}{R_i} \frac{4.76}{9.52} = \frac{1}{2} \frac{R_f}{R_i}. \]

\[ +2 \]
So \[ R_i = 10 \text{ k}\Omega, \quad C_i = 0.476 \mu\text{F}. \] \[ \Rightarrow R_f = 84.4 \text{ k}\Omega, \quad C_f = 2.26 \text{ nF}. \]

\[ \Rightarrow R_f = 2 \times (4.27) R_i. \]
2. (25 points) On the next page you will find the phase Bode plot for a certain circuit. There are no breakpoints outside the frequency range shown. The magnitude Bode plot (not shown) increases with frequency at 20 dB/dec at very low frequencies, and has a constant value of 35 dB at $\omega = 2,000$ rad/s. Find the transfer function for this circuit.

$\textbf{a)}$ Drop of $90^\circ$/dec at $\omega = 2,000$ rad/s $\Rightarrow$ double pole at $\omega = 20$ rad/s.

$\textbf{b)}$ Drop reduced to $45^\circ$/dec at 40 rad/s $\Rightarrow$ zero at $\omega = 400$ rad/s.

$\omega = 200$ rad/s : effect of double pole ends.

$\textbf{c)}$ Increase of $45^\circ$/dec at $\omega = 10^3$ rad/s $\Rightarrow$ zero at $\omega = 10^4$ rad/s.

$\omega = 4000$ rad/s : zero ends

$\omega = 10^5$ rad/s : zero ends.

The $j\omega$ in the numerator accounts for the 20 dB/dec slope indicated in the magnitude Bode plot.

Also, $|T(j\omega)|_{\omega = 2000}$ rad/s = 35 dB $\Rightarrow$ $|T(j\omega)|_{\omega = 2}$ kHz = 56.2

But $|T(j\omega)|_{\omega = 2000}$ rad/s = $1.04 \times 10^4$ $K = 56.2$

$\Rightarrow K = 5.4 \times 10^{-3}$

Finally, we note that $\angle T(j\omega) \rightarrow 90^\circ$, but the plot indicates $\angle T(j\omega) \rightarrow -90^\circ$, so there must also be a ' - ' sign in $T(j\omega)$. 5
3. (25 points) For the circuit below, do the following.

i. Choose component values to produce the output

\[ v_o(t) = -2.5 \sin(\omega t) + V_{\text{offset}} \quad [V] \]

where \( V_{\text{offset}} \) is a dc offset voltage that ranges from +4 V to -4 V, and the potentiometer is a 10-turn, 20 kΩ potentiometer.

ii. The 10 kΩ potentiometer is set to 4 turns from its end value, with the smaller resistance above the wiper (as it is oriented in the figure). Write an expression for the output voltage \( v_o(t) \).

i) If we turn the pot all the way to the top, we will have (ignoring \( V_s \) input):

\[ V_o = \frac{R_f}{R_1} \times 10 \]

In this case, we need \( V_o = -4 \) V, \( \Rightarrow \frac{R_f}{R_1} = 0.4 \).
choosing $R_f = 10\, k\Omega \Rightarrow R_1 = 25\, k\Omega$

To get an ac component of $-2.5\, V$ (amplitude), we will need $R_2 = 4.1\, k\Omega$

At 4 turns, which is $0.4 \times 10\, k\Omega = 8\, k\Omega$, we have:

We again ignore $V_s$ and focus on the ac component.

$R_x = 8.1\, k\Omega \quad R_y = 12\, k\Omega$

Taking a Thevenin equivalent, we have (dc component)

$R_{th} = 8.1\, k\Omega / 12\, k\Omega = 4.8\, k\Omega \quad V_{th} = 20^\circ \frac{12}{20} -10 = 2\, V.$

$V_0 = \frac{-10}{29.8} \cdot 2 = -0.67\, V.$

$\therefore \ V_0 = -0.67 - 2.5\sin(\omega t)$
4. (30 points) For the circuit below, do the following.

i. Find the Thevenin equivalent circuit with respect to the terminals of the diode.
ii. The diode $i_d - v_d$ characteristics are shown in the graph on the next page. On this graph, plot the load line for the circuit.
iii. Based on your plot, estimate all possible operating points, i.e., the current and voltage values taken by the diode.

This looks like the negative impedance converter but it's not identical. Let's do the analysis:

\[ i_T' = \frac{V_T + V_T - V_T}{R} \]

\[ V_T = V_T (1 + \frac{R}{R}) = 2V_T \]

\[ \Rightarrow i_T' = \frac{-V_T}{2R} \]

KVL: \[ V_T + \frac{R}{2} i_T' - V_T = 0 \]

\[ V_T = V_T - \frac{R}{2} i_T' \]

\[ \Rightarrow i_T' = \frac{-V_T}{2R} + \frac{1}{4} l_T' \]

\[ \frac{V_T}{l_T} = \frac{-2}{3} R = R_{TH} \]
So we have:

\[ i_d = \frac{v_d + 0.8}{3R} = \frac{2}{3R} v_d + \frac{1.6}{3R} \]  

So this is a straight line with slope \( \frac{2}{3R} \).

For \( i_d = 0 \), \( v_d = -0.8 \) Volt; for \( v_d = 0 \), \( i_d = \frac{2}{3R} = 13.3 \mu A \). These points are plotted above. The load line implies two possible operating points:

\[ i_d = -5 \mu A, v_d = -1.6 \text{ Volt}; i_d = 19 \mu A, v_d = 0.3 \text{ Volt}. \]